VIII. *On the progressive improvements made in the efficiency of steam engines in Cornwall, with investigations of the methods best adapted for imparting great angular velocities.* By Davies Gilbert, *Esq. President.*

Read March 4, 1830.

In the year 1827, some observations I had made on steam engines were honoured by a place in the Philosophical Transactions. I am therefore induced to lay before the Society further particulars illustrative of the progress by which that most important machine has reached its actual high state of improvement. On a subject of less magnitude I should not have presented to the Society a mere collection of matter in detail, unconnected by any general arrangement of the facts: but every thing appears to me of great interest that bears on the history of an invention that has continually advanced towards perfection by the aid of chemical, mechanical, and mathematical sciences; an invention that has already altered and improved the condition of mankind; and seems destined to produce consequences the most beneficial to civilized society, by extending the dominion of intellect over muscular power and brute force. I am moreover desirous of preserving information derived from documents which have never yet passed out of private hands, and are consequently liable to be lost or destroyed.

For all practical purposes the steam engine must be considered as originating with Mr. Newcomen; the introduction of a moveable diaphragm between the active power and the vacuum or less elastic medium, being essential to the very principle of the machine as a moving power.

Mr. Newcomen's engines were brought into Cornwall very early in the last century, where they immediately superseded the laborious method of drawing water by human exertions, applied through the simple medium of a chain pump, similar in construction to those at present used on board large ships. So inartificial, indeed, was the machinery in mines at this comparatively recent period, that I well remember an individual who used to boast of his having assisted in constructing the first Whim, or contrivance for applying the strength of horses to the lifting of weights, that was seen in the peninsula west of Hayle and of St. Michael's Mount.

The only material improvement ever made to Mr. NEWCOMEN's engine, had, I apprehend, been effected previously to its introduction into Cornwall, which consisted in the automatic opening and shutting of the valves. It consequently remained in nearly the same state up to the time when a new æra commenced from the appearance of Messrs. BOULTON and WATT.

The use of Mr. WATT's engine, secured to him by a patent in 1769, and extended by Parliament to the year 1800, was offered on terms reasonable in themselves, and fairly growing out of the subject. These were, payments of one third part of all the savings in fuel, to be estimated by a comparison of the new engines with those on the former construction executing equal quantities of work.

Experiments on a large scale were accordingly instituted in the summer of 1778 on two atmospheric engines, then supposed to be the most perfect of any at work in Cornwall; conducted by Messrs. BOULTON and WATT themselves on one part, and by the engineers and managers of mines on the other, assisted by some of the most respectable gentlemen of the county, who were interested either as proprietors of the land, or as adventurers in the mines.

The following is a copy of their Report:

"Poldice Mines, October 30, 1778.

"We the under subscribers having carefully examined the books of the mine touching the consumption of coals of the two eastern engines during the months of August and September of this present year, and attended to the working of the engines during those months, do hereby certify, that the said two eastern engines did consume in 61 days of those two months 220 weys of coal, each wey being 64 Winchester coal bushels, which amount to 14'080 bushels in 61 days.

"We do also certify, that the said two engines together do work pumps in four lifts which are 17 inches diameter in the working barrels, and the whole depth from whence these pumps drew water to the adit is 58 fathoms.

"We further certify, that the said engines did during those months of August and September last, work the said pumps at the rate of 6 strokes of $5\frac{1}{2}$ feet long each in every minute, which amounts to 8640 strokes per 24 hours.

"We have also made an accurate calculation, by which it appears that when the new fire-engine to be erected by Messrs. BOULTON and WATT is completed, and actually works a pump of the same depth of 58 fathoms and 17 inches in diameter at the rate of 6 strokes of $5\frac{1}{2}$ feet long each in a minute, and consequently making 8640 strokes per 24 hours, it will draw a quantity of water equal to that now drawn by both the present engines, and consequently whatever smaller quantity of coals it uses than 14'080 bushels for 61 days when going at the rate of 6 strokes per minute, will be the real savings in fuel occasioned by the said new engine at that rate of going.

(Signed) "JAMES WATT.
"MATTHEW BOULTON.
"H. HAWKINS TREMAYNE.
"RICHARD WILLIAMS.
"JOHN WILLIAMS.
"THOMAS BROWN."

A pound avoirdupoise lifted through one foot had not at that period been established as the dynamic unit.

The product of pounds raised, and of the number of feet through which they are lifted in a given time, divided by the number of bushels of coal (supposed to weigh 84 pounds) burnt in the same interval, give what is now termed the duty of the engine, and afford a perfect criterion of its comparative merit.

The most convenient method of forming this estimate is by multiplying together, the diameter in inches squared, of the lifting box, or of the plunger piston, the height of the lifts in fathoms, and 2.04 (log. 3.3095101) the weight in pounds avoirdupoise of a cylinder of water one inch in diameter and six feet long, the product gives the weight.

This, multiplied by the length of the stroke in feet, and by the number of strokes in a given time, and finally divided by the number of bushels of coal consumed, will give the duty.

In the certified case,

Inches in the diameter of the lifting box 17, sq. 289 . . .	Log. 2.4608978
Length of the column in fathoms 58	1.7634280
Constant multiplier 2.04	0.3095101
Weight in pounds 34185	4.5338359
Length of lifts 6 feet × $5\frac{1}{2}$ the number in a minute = 33, 33 multiplied by 1440 = 47520, and this number × by 61 = 2'898720	6.4622063
Pounds into feet during the whole period of 61 days 99092'800000	10.9960422
Bushels of coal consumed in the 61 days 14.080	4.1486027
The duty 7'037800	6.8474395

If the quantity of water lifted at each stroke is required in imperial gallons; square as before the diameter of the lifting box or plunger in inches, multiply by the length of the stroke in feet, and by 0.034 (the fractional part of an imperial gallon in a cylinder one inch in diameter and a foot long—the logarithm 8.5313588): the product will be the gallons raised.

Diameter in inches squared as before	Log. 2.4608978
6 feet	0.7781513
Constant multiplier 0.034	8.5313588
Number of gallons 58.94	1.7704079
$5\frac{1}{2}$ strokes in a minute, × minutes in 61 days 87'840 = 483.120	5.6840550
Imperial gallons in 61 days = 28'475000	7.4544629

Each separate engine on the new construction underwent a constant comparison, during the whole time of its continuing in action, with the duty of the two standard atmospheric engines as reported by the committee.

The diameter of the various lifting boxes or plungers, the length of the lifts or columns of water, and the lengths of the strokes, were matters of common notoriety; and the number of times moved by each engine in a given period was ascertained by a contrivance denominated a Counter, placed on the great beam; this apparatus involves a series of wheels and pinions set in motion by a weight rolling in each direction, and acting through the medium of an escapement similar to that of a clock.

In the year 1793, that is fifteen years afterwards, an account was taken of the work performed by seventeen engines on Mr. WATT's construction, then working in Cornwall. This statement is not verified in the authentic manner of the former; but there do not appear to be any reasons for doubting of its correctness. The average duty was 19'569000 pounds of water raised one foot high, by the consumption of one bushel of coal; exceeding the standard experiment on the two atmospheric engines in the proportion of 2.78 to 1, or nearly as $2\frac{3}{4}$ to 1. So that work requiring by the atmospheric engines 278 bushels of coal, would be performed by Mr. WATT's steam engine by 100 bushels, consequently 178 would be saved. One third part of these must have been paid to Messrs. BOULTON and WATT as patentees, leaving a clear gain to the mine of 118 bushels, being more than the quantity consumed.

Some years later, disputes took place as to the real performance of Mr. WATT's engines, and a reference was agreed on between the parties to five individuals, of whom I had the honour to be one; and in May 1798 returns were made by the agents in various mines, of all the particulars respecting twenty-three engines, from which I then deduced their respective duties for the information of the referees.

It will not be necessary to trouble the Society with more of these particulars than the diameter of the cylinder, whether it worked single or condensed, both above and below, and the final result or duty in millions of pounds lifted one foot high by the consumption of one bushel of coals.

	Diameter of Cylinder in inches.	Duty.	Observations.
1	20	10'015000	
2	21	16'385000	
3	45	29'668000	It was believed at the time that some inaccuracy must have occurred in the communications respecting these two engines.
4	36	28'212000	
Double 5	42	18'193000	On the same mine. The length of strokes in all but one, six feet; in that, eight feet. Average duty of the whole, 15'985000.
6	63	15'190000	
Double 7	45	15'180000	
8	45	15'571000	
9	45	15'090000	
10	45	14'384000	
Double 11	42	18'740000	
12	42	15'532000	
13	36	18'465000	
14	—	12'226000	The diameter of the cylinder not returned.
15	30	14'050000	
16	20	12'366000	
Double 17	$14\frac{3}{4}$	6'097000	
18	30	13'931000	
19	28	19'739000	
20	36	24'514000	
21	21	13'215000	
22	20	15'034000	
23	48	27'503000	Supposed to be the best engine.
		17'671000	The general Average in 1798.

It may be observed that the average duty was here somewhat less, than it had been found in 1793, confirmatory of an opinion generally entertained that the steam engines had deteriorated from the time of Mr. WATT's quitting his residence near Redruth.

The principles and even the mechanism of Mr. WATT's engines have remained unaltered since their first introduction, unless a change in the precise periods of opening and closing the valves could be considered a variation. But to such an extent has the economy of fuel been carried by the use of steam at a high degree of temperature and consequently of pressure, usually from fifty to sixty inches of mercury above the atmosphere, by extending the expansive action to two thirds or even to three quarters of the whole descent of the piston, by making small fire-places, with sharper drafts, in iron tubes surrounded by the water of the boiler, by more effectually preventing the escape of heat, by enlarging the engines themselves, and perhaps by executing the work with superior accuracy, that in the monthly return of duty performed in Cornwall by the steam engines in December 1829,—the best engine with a cylinder of 80 inches did 75'628000, exceeding the duty performed in 1795 in the proportion of 3.865 to 1, or as 27 : 7 nearly: and exceeding the standard atmospheric engines of 1778 by 10.75 : 1. But subject as in former times to a great variation between different engines apparently similar in all respects; the average being about forty-one million and a half.

Greatly as we are indebted to Mr. WATT for his improvement of the steam engine used in exhausting water from mines, our obligations to him are still greater for originating and carrying almost to a state of perfection, the application of steam as a moving power to machinery, in all the complicated and varied uses of mechanical inventions in this country.

In effecting this most important object, the double engine was first brought into use, the extremely ingenious contrivance for producing parallel motion was invented, and the principle of centrifugal force enabled an apparatus called a Governor to regulate a supply of steam inversely proportionate to the velocity which might at any instant be acquired; and the use of fly-wheels, perfectly understood in theory, became subservient to the regularity of motion, and to the gigantic efforts of our most ponderous machinery. We owe further to Mr. WATT the introduction, at least into general use, of what is termed bevelled geer.

When wheels acting on each other by means of teeth have their axes parallel, the teeth however curved on themselves, must obviously be parallel also on their line of contact. But when the axes are inclined, the line of contact between the teeth should then be directed to the point where the two axes would intersect, thus assimilating their action to that of a cone revolving round a centre on a circular plane.

Having on various occasions had my attention drawn to the consideration of machines forced into rapid action by great powers moving at a comparatively slow rate, I have been induced to make endeavours for ascertaining the amount of friction in several instances.

The mode adopted has been to impart equal velocities to the machine performing the work for which it was destined, and in a state disincumbered from all extraneous impediments, and then to compare the forces applied under these different circumstances. The media of communication being in all cases axes with wheels, impelled by teeth, or cogs.

In some instances the friction was found to equal two-thirds of the whole resistance, in others a half, and in some few instances one-third: evidently becoming less as the wheels were increased in size; and always greatly diminished by the introduction of an additional axis for multiplying the angular velocity. In similar machines recently constructed on the best principles, friction is said not to exceed a tenth.

With the hope of reducing the amount of this great impediment to the useful application of motive force, I have been led to consider what would be the most proper form of teeth or cogs, and by how many intermediate steps a given increase of angular velocity might be most advantageously effected.

It is quite clear, with respect to cogs or teeth, that to give them theoretical perfection, they should be so formed as to communicate an equable velocity from the driving to the driven wheel; and at the same time to roll upon themselves free from any sliding, the cause of friction and of abrasion.

Either of these properties may be separately obtained; but the two are utterly incompatible, except at the limit where teeth disappear and the wheels themselves are in contact.

For producing equable velocities.

The involute from a circle, receives continually an accession of length to its radius of curvature equal to the development of the generative arc. Consequently, if involutes are formed in opposite directions from two circles of magnitudes inversely proportionate to the angular velocities of their respective wheels, the extremities of the two radii of curvature will always remain in contact, forming together a tangent to the evolute circles crossing the line of the centres. The angular velocities must therefore be uniform, while the surfaces will slide on each other, and create a friction proportionate to the difference of length between the radii of curvature.

For avoiding friction between the cogs or teeth.

As logarithmic spirals preserve always a constant angle between the ordinate and the curve, if two similar logarithmic spirals act against each other, they must continue to roll without friction, the ordinates remaining in contact on the line of the centres, but causing angular velocities in the inverse proportion of those ordinates.

It is plain therefore that the two properties are incompatible, since the loci of contact between the curves producing them are in different lines. But since the involutes may be generated from circles indefinitely small, without reference to the size of the wheels, and the logarithmic spirals may be inclined at any angle: at their ultimate limit the lines of the centres and of the tangents will coincide, producing equable velocities and avoiding friction; but at this point the teeth or cogs disappear, and the wheels revolve with a contact of their peripheries.

As cogs become oblique, their contact produces an increase of pressure with augmented friction on the axes: but I am induced to believe, from the results of such trials as I have been enabled to make, as well as from theory, that an uniform angular motion is the great object to be sought, especially in ponderous machinery, and above all where fly-wheels are applied. The expedient usually resorted to, of making the teeth small and consequently many in number, renders attention to any precise curvature less necessary; but this subdivision is limited by want of strength; and I would venture to recommend the involute form, producing an equability of angular motion.

When angular velocities are to be multiplied in large machines, the effect can scarcely be produced in any other way than by the usual one, of wheels unequal in diameter and consequently in the numbers of their teeth.

The question then to be resolved is this:

By the interposition of how many such pairs of wheels can the ultimate angular velocity be most advantageously produced?

Wishing to obtain a solution accurate to no further degree than what may be sufficient for the guidance of practical experiments, I omit to take any separate account of the friction caused by the teeth of the wheels as distinct from that of their axes, and I shall consider the wheels and axes throughout the train in a constant ratio to each other, although as the actual pressure becomes less, the axes may be reduced in size, and thereby friction diminished.

When two forces act on an axis from the peripheries of two wheels attached to it; one for receiving motion, and the other for conveying it on through the train, the pressure on the axis may be the sum of the two forces, their difference, or any intermediate quantity according to the positions of the points communicating with the preceding and with the subsequent wheels; the pressure is most usually their sum, as convenience generally requires that the power should be received and carried forwards at opposite points.

Let the diameters of the two wheels be as $1 : d$, then will the amount of pressure on the axis be $d + 1$. Assuming the axis and wheels next in succession to be similar in all respects, the pressure on them will be less than on the former in proportion to the increased angular velocity, but the prime mover having the disadvantage of leverage in the same proportion, the retarding effect of friction will be precisely equal in both; whence it is obvious, that the same ratio should continue throughout all the series, or that the multiplication of angular velocity should proceed in a geometrical proportion.

Let then the whole increase of angular velocity be represented by a, and let the number of axes employed be x. Then in the usual mode of applying the wheels the friction on each axis will be $a^{\frac{1}{x}} + 1$. And the friction of the whole of the axes will be $(a^{\frac{1}{x}} + 1)\,x$. To find when this is a maximum, let A = the natural logarithms of a. Then will the fluxion of $(a^{\frac{1}{x}} + 1)\,x$ be

$$- \mathrm{A} \,.\, a^{\frac{1}{x}} . x^{-1} \dot{x} + a^{\frac{1}{x}} \dot{x} + \dot{x} \text{ when this is put } = 0$$

$$x = \frac{\mathrm{A}}{1 + a^{-\frac{1}{x}}}$$

By approximating it will be found that when

$a = 120$ x or the number of axes $= 3.745$, and $(a^{\frac{1}{x}} + 1).x$ or the friction 17.9 instead of 121 if x were unity and the whole angular velocity communicated at once, about one seventh part.

$a = 100$ $x = 3.6$ friction 15.64 about 10 parts in 64 of the whole, as above.

$a = 40$ $x = 2.88$ friction 13.25, about one third part of 41.

When $a = 3.59$ the minimum of friction falls on a single axis.

$a = 12.85$ the minimum of friction is on two axes.

$a = 46.3$ the minimum of friction is on three axes.

$a = 166.4$ the minimum falls on four axes.

In practice it is obvious that x must represent a whole number.

Let $a = 64$ x may be either two or three.

If $x = 2$ the friction will be 18.
3 the friction will be 15.

If the wheels be so arranged that the resultant of the two forces on the axes equal the larger in magnitude, x becomes = to A, and the amount of friction $a^{\frac{1}{A}} \times A$ but $a^{\frac{1}{A}} =$ the radix of the natural system of logarithms; consequently the friction in this case will be $e \times A$.

If the points of the two wheels receiving and communicating motion are placed in the same right line parallel to the axes, the general expression becomes $x = \frac{A}{1 - a^{-\frac{1}{x}}}$ evidently giving to x an infinite magnitude: but this can indicate no more in practice than the advantage of so arranging the communications of motion when it can possibly be done.

As fly-wheels are of essential use in preserving uniform velocities, or for accumulating power in almost all rotatory movements produced from those that alternate, and as the power of the centrifugal force has sometimes exceeded the cohesion even of iron, I shall conclude this paper with an adaptation of a well-known theorem to common use.

Let $r =$ the radius of a wheel expressed in feet.
$v =$ the velocity of the rim where all the weight is supposed to be accumulated, expressed in feet in a second.
$s =$ the space in feet through which a body descends in one second 16.0899; log. 1.2065541.
F = the centrifugal force.

$$\text{Then } F = \frac{v^2}{2 r . s}$$

Let $n =$ the number of revolutions in a second.
$c =$ the periphery of the circle to diameter unity = 3.14159.
log. 0.4971499.

$$\text{Then } F = r . n^2 \times 1.2268 \text{ (log. } 0.0887757).$$

Consequently for an approximate value,

The centrifugal force = the radius × number of revolutions in a second squared × 1.2.

If N = the number of revolutions in a minute, the product found as above must be divided by $60 \times 60 = 3600$; consequently $F = r N^2 \times 0.00034078$ (log. 6.5324732.)

For an approximate value,

The centrifugal force = the radius $\times$ number of revolutions in a minute squared $\times$ 34 and cutting off five places of figures.